

## Electric Circuits



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This book is intended to be an introductory text on the subject of electric circuits. It provides simple explanations of the basic concepts, followed by simple examples and exercises. When necessary, detailed derivations for the main topics and examples are given to help readers understand the main ideas. MATLAB is a tool that can be used effectively in Electric Circuits courses. In this text, MATLAB is integrated into selected examples to illustrate its use in solving circuit problems. MATLAB can be used to check the answers or solve more complex circuit problems. This text is written for a two-semester sequence or a three-quarters sequence on electric circuits.

## Suggested Course Outlines

The following is a list of topics covered in a typical Electric Circuits courses, with suggested course outlines.

## ONE-SEMESTER OR -QUARTER COURSE

If Electric Circuits is offered as a one-semester or one-quarter course, Chapters 1 through 12 can be taught without covering, or only lightly covering, sections $1.6,2.10,2.11,3.6,4.7$, $5.6,5.7,5.8,6.7,7.6,7.7,8.8,8.9,9.9,9.10,10.12,11.7,12.5,12.6$, and 12.7 .

## TWO-SEMESTER OR -QUARTER COURSES

For two-semester Electric Circuit courses, Chapters 1 through 8, which cover dc circuits, op amps, and the responses of first-order and second-order circuits, can be taught in the first semester. Chapters 9 through 20, which cover alternating current (ac) circuits, Laplace transforms, circuit analysis in the $s$-domain, two-port circuits, analog filter design and implementation, Fourier series, and Fourier transform, can then be taught in the second semester.

## THREE-QUARTER COURSES

For three-quarter Electric Circuit courses, Chapters 1 through 5, which cover dc circuits and op amps, can be taught in the first quarter; Chapters 6 through 13, which cover the responses of first-order and second-order circuits and ac circuits, can be taught in the second quarter, and Chapters 14 through 20, which cover Laplace transforms, circuit analysis in the $s$-domain, two-port circuits, analog filter design and implementation, Fourier series, and Fourier transform, can be taught in the third quarter.

Depending on the catalog description and the course outlines, instructors can pick and choose the topics covered in the courses that they teach. Several features of this text are listed next.

## Features

After a topic is presented, examples and exercises follow. Examples are chosen to expand and elaborate the main concept of the topic. In a step-by-step approach, details are worked out to help students understand the main ideas.

In addition to analyzing RC, RL, and RLC circuits connected in series or parallel in the time domain and the frequency domain, analyses of circuits different from RC, RL, and RLC circuits and connected other than in series and parallel are provided. Also, general input signals that are different from unit step functions are included in the analyses.

In the analog filter design, the specifications of the filter are translated into its transfer function in cascade form. From the transfer function, each section can be designed with appropriate op amp circuits. The normalized component values for each section are found by adopting a simplification method (equal R equal C or unity gain). Then, magnitude scaling and frequency scaling are used to find the final component values. The entire design procedure, from the specifications to the circuit design, is detailed, including the PSpice simulation used to verify the design.

Before the discussion of Fourier series, orthogonal functions and the representation of square integrable functions as a linear combination of a set of orthogonal functions are introduced. The set of orthogonal functions for Fourier series representation consists of cosines and sines. The Fourier coefficients for the square pulse train, triangular pulse train, sawtooth pulse train, and rectified sines and cosines are derived. The Fourier coefficients of any variation of these waveforms can be found by applying the time-shifting property and finding the dc component.

MATLAB can be an effective tool in solving problems in electric circuits. Simple functions such as calculating the equivalent resistance or impedance of parallel connection of resistors, capacitors, and inductors; conversion from Cartesian coordinates to polar coordinates; conversion from polar coordinates to Cartesian coordinates; conversion from the wye configuration to delta configuration; and conversion from delta configuration to wye configuration provide accurate answers in less time. These simple functions can be part of scripts that enable us to find solutions to typical circuit problems.

The complexity of taking the inverse Laplace transforms increases as the order increases. MATLAB can be used to solve equations and to find integrals, transforms, inverse transforms, and transfer functions. The application of MATLAB to circuit analysis is demonstrated throughout the text when appropriate. For example, after finding inverse Laplace transforms by hand using partial fraction expansion, answers from MATLAB are provided as a comparison.

Examples of circuit simulation using OrCAD PSpice and Simulink are given at the end of each chapter. Simulink is a tool that can be used to perform circuit simulations. In Simulink, physical signals can be converted to Simulink signals and vice versa. Simscapes include many blocks that are related to electric circuits. Simulink can be used in computer assignments or laboratory experiments.

The Instructor's Solution Manual for the exercises and end-of-chapter problems is available for instructors. This manual includes MATLAB scripts for selected problems as a check on the accuracy of the solutions by hand.

## Overview of Chapters

In Chapter 1, definitions of voltage, current, power, and energy are given. Also, independent voltage source and current source are introduced, along with dependent voltage sources and current sources.

In Chapter 2, nodes, branches, meshes, and loops are introduced. Ohm's law is explained. Kirchhoff's current law (KCL), Kirchhoff's voltage law (KVL), the voltage divider rule, and the current divider rule are explained with examples.

In Chapter 3, nodal analysis and mesh analysis are discussed in depth. The nodal analysis and mesh analysis are used extensively in the rest of the text.

Chapter 4 introduces circuit theorems that are useful in analyzing electric circuits and electronic circuits. The circuit theorems discussed in this chapter are the superposition
principle, source transformations, Thévenin's theorem, Norton's theorem, and maximum power transfer.

Chapter 5 introduces op amp circuits. Op amp is a versatile integrated circuit (IC) chip that has wide-ranging applications in circuit design. The concept of the ideal op amp model is explained, along with applications in sum and difference, instrumentation amplifier, and current amplifier. Detailed analysis of inverting configuration and noninverting configuration is provided.

In Chapter 6, the energy storage elements called capacitors and inductors are discussed. The current voltage relation of capacitors and inductors are derived. The energy stored on the capacitors and inductors are presented.

In Chapter 7, the transformation of RC and RL circuits to differential equations and solutions of the first-order differential equations to get the responses of the circuits are presented. In the general first-order circuits, the input signal can be dc, ramp signal, exponential signal, or sinusoidal signal.

In Chapter 8, the transformation of series RLC and parallel RLC circuits to the secondorder differential equations, as well as solving the second-order differential equations to get the responses of the circuits are presented. In the general second-order circuits, the input signal can be dc, ramp signal, exponential signal, or sinusoidal signal.

Chapter 9 introduces sinusoidal signals, phasors, impedances, and admittances. Also, transforming ac circuits to phasor-transformed circuits is presented, along with analyzing phasor transformed circuits using KCL, KVL, equivalent impedances, delta-wye transformation, and wye-delta transformation.

The analysis of phasor-transformed circuits is continued in Chapter 10 with the introduction of the voltage divider rule, current divider rule, nodal analysis, mesh analysis, superposition principle, source transformation, Thévenin equivalent circuit, Norton equivalent circuit, and transfer function. This analysis is similar to the one for resistive circuits with the use of impedances.

Chapter 11 presents information on ac power. The definitions of instantaneous power, average power, reactive power, complex power, apparent power, and power factor are also given, and power factor correction is explained with examples.

As an extension of ac power, the three-phase system is presented in Chapter 12. The connection of balanced sources (wye-connected or delta-connected) to balanced loads (wye-connected or delta connected) are presented, both with and without wire impedances.

Magnetically coupled circuits, which are related to ac power, are discussed in Chapter 13. Mutual inductance, induced voltage, dot convention, linear transformers, and ideal transformers are introduced.

The Laplace transform is introduced in Chapter 14. The definition of the transform, region of convergence, transform, and inverse transform are explained with examples. Various properties of Laplace transform are also presented with examples.

The discussion on Laplace transform is continued in Chapter 15. Electric circuits can be transformed into an $s$-domain by replacing voltage sources and current sources to the $s$-domain and replacing capacitors and inductors to impedances. The circuit laws and theorems that apply to resistive circuits also apply to $s$-domain circuits. The time domain signal can be obtained by taking the inverse Laplace transform of the $s$-domain representation. The differential equations in the time domain are transformed to algebraic equations in the $s$-domain. The transfer function in the $s$-domain is defined as the ratio
of the output signal in the $s$-domain to the input signal in the $s$-domain. The concept of convolution is introduced with a number of examples. Also, finding the convolution using Laplace transforms are illustrated in the same examples. Plotting the magnitude response and phase response of a circuit or a system using the Bode diagram is introduced.

The first-order and the second-order analog filters that are building blocks for the higher-order filters are presented in Chapter 16. The filters can be implemented by interconnecting passive elements consisting of resistors, capacitors, and inductors. Alternatively, filters can be implemented utilizing op amp circuits. Sallen and Key circuits for implementing second-order filters are discussed as well, along with design examples.

The discussion on analog filter design is extended in Chapter 17. A filter is designed to meet the specifications of the filter. The transfer function that satisfies the specification is found. From the transfer function, the corner frequency and $Q$ value can be found. Then, the normalized component values and scaled component values are found. PSpice simulations can be used to verify the design.

Orthogonal functions and the representation of signals as a linear combination of a set of orthogonal functions are introduced in Chapter 18. If the set of orthogonal functions consists of harmonically related sinusoids or exponential functions, the representation is called the Fourier series. Fourier series representation of common signals, including the square pulse train, triangular pulse train, sawtooth waveform, and rectified cosine and sine, are presented in detail, with examples. The derivation and application of the time-shifting property of Fourier coefficients are provided. In addition, the application of the Fourier series representation in solving circuit problems are presented, along with examples.

As the period of a periodic signal is increased to infinity, the signal becomes nonperiodic, the discrete line spectrums become a continuous spectrum, and multiplying the Fourier coefficients by the period produces the Fourier transform, as explained in Chapter 19. Important properties of the Fourier transform, including time shifting, frequency shifting, symmetry, modulation, convolution, and multiplication, are introduced, along with interpretation and examples.

Two-port circuits are defined and analyzed in Chapter 20. Depending on which of the parameters are selected as independent variables, there are six different representations for two-port circuits. The coefficients of the representations are called parameters. The six parameters ( $z, y, h, g, A B C D, b)$ for two-port circuits are presented along with examples. The conversion between the parameters and the interconnection of parameters are provided in this chapter.

## Instructor Resources

Cengage Learning's secure, password-protected Instructor Resource Center contains helpful resources for instructors who adopt this text. These resources include Lecture Note Microsoft PowerPoint slides, test banks, and an Instructor's Solution Manual, with detailed solutions to all the problems from the text. The Instructor Resource Center can be accessed at https://login.cengage.com.

## MindTap Online Course

Electric Circuits is also available through MindTap, Cengage Learning's digital course platform. The carefully crafted pedagogy and exercises in this textbook are made even more effective by an interactive, customizable eBook, automatically graded assessments, and a full suite of study tools.

| $<$ | CHAPTER 6: CAPACIORS AND INDUCTORS |
| :---: | :---: |
| ¢ | Chapter 6: Capacitors and Inductors <br> Introduction - Capacitors - Series and Parallel Connection of Capacitors - Op Amp Integrator and Op Amp Differentiator - Inductors - Series and Parallel Connection of Inductors - PSpice and Simulink - Summary |
| ( | Chapter 6 Lecture <br> Watch this lecture on Capacitors and Inductors. |
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## Voltage, Current, Power, and Sources

### 1.1 Introduction

The seven base units of the International System of Units (SI), along with derived units relevant to electrical and computer engineering, are presented in this chapter. The definitions of the terms voltage, current, and power are given as well.

A voltage source with voltage $V_{s}$ provides a constant potential difference to the circuit connected between the positive terminal and the negative terminal. A current source with current $I_{s}$ provides a constant current of $I_{s}$ amperes to the circuit connected to the two terminals. If the voltage from the voltage source is constant with time, the voltage source is called the direct current (dc) source. Likewise, if the current from the current source is constant with time, the current source is called the $d c$ source. If the voltage from the voltage source is a sinusoid, the voltage source is called alternating current (ac) voltage source. Likewise, if the current from the current source is a sinusoid, the current source is called the ac current source.

The voltage or current on the dependent sources depends solely on the controlling voltage or controlling current. Dependent sources are introduced along with circuit symbols.

The elementary signals that are useful throughout the text are introduced next. The elementary signals are Dirac delta function, step function, ramp function, rectangular pulse, triangular pulse, and exponential decay.

### 1.2 International System of Units

The International System of Units (SI) is the modern form of the metric system derived from the meter-kilogram-second (MKS) system. The SI system is founded on seven base units for the seven quantities assumed to be mutually independent. Tables 1.1-1.6, which
give information on the SI system, come from the NIST Reference on Constants, Units, and Uncertainty (http://physics.nist.gov/cuu/Units/units.html), the official reference of the National Institute of Standards and Technology.

A meter is defined as the length of a path traveled by light in a vacuum during a time interval of $1 / 299,792,458\left[\left(\approx 1 /\left(3 \times 10^{8}\right)\right]\right.$ of a second.

A kilogram is equal to the mass of the international prototype of the kilogram.

| TABLE 1.1 | Base Quantity | Name | Symbol |
| :---: | :--- | :--- | :--- | :--- |
| SI Base Units. | Length | meter | m |
|  | Mass | kilogram | kg |
|  | Time | second | s |
|  | Electric current | ampere | A |
|  | Thermodynamic temperature | kelvin | K |
|  | Amount of a substance | mole | mol |
|  | Luminous intensity | candela | cd |


| TABLE 1.2 | Derived Quantity | Name | Symbol |
| :---: | :---: | :---: | :---: |
| Examples of SI Derived Units. | Area | square meter | $\mathrm{m}^{2}$ |
|  | Volume | cubic meter | $\mathrm{m}^{3}$ |
|  | Speed, velocity | meter per second | $\mathrm{m} / \mathrm{s}$ |
|  | Acceleration | meter per second squared | $\mathrm{m} / \mathrm{s}^{2}$ |
|  | Wave number | reciprocal meter | $\mathrm{m}^{-1}$ |
|  | Mass density | kilogram per cubic meter | $\mathrm{kg} / \mathrm{m}^{3}$ |
|  | Specific volume | cubic meter per kilogram | $\mathrm{m}^{3} / \mathrm{kg}$ |
|  | Current density | ampere per square meter | $\mathrm{A} / \mathrm{m}^{2}$ |
|  | Magnetic field strength | ampere per meter | A/m |
|  | Luminance | candela per square meter | $\mathrm{cd} / \mathrm{m}^{2}$ |


| TABLE 1.3 |
| :---: | :--- | :--- | :--- | :--- | | SI Derived Units <br> with Special <br> Names and <br> Symbols. | Derived Quantity | Plane angle | Name |
| :---: | :--- | :--- | :--- |


| TABLE 1.4 | Derived Quantity | Name | Symbol |
| :---: | :---: | :---: | :---: |
| Examples of SI Derived Units with Names and Symbols (Including Special Names and Symbols.) | Dynamic viscosity | Pascal second | $\mathrm{Pa} \cdot \mathrm{s}$ |
|  | Moment of force | newton meter | $\mathrm{N} \cdot \mathrm{m}$ |
|  | Surface tension | newton per meter | $\mathrm{N} / \mathrm{m}$ |
|  | Angular velocity | radian per second | $\mathrm{rad} / \mathrm{s}$ |
|  | Angular acceleration | radian per second squared | $\mathrm{rad} / \mathrm{s}^{2}$ |
|  | Heat flux density, irradiance | watt per square meter | $\mathrm{W} / \mathrm{m}^{2}$ |
|  | Thermal conductivity | watt per meter kelvin | $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K})$ |
|  | Energy density | joule per cubic meter | $\mathrm{J} / \mathrm{m}^{3}$ |
|  | Electric field strength | volt per meter | $\mathrm{V} / \mathrm{m}$ |
|  | Electric charge density | coulomb per cubic meter | $\mathrm{C} / \mathrm{m}^{3}$ |
|  | Electric flux density | coulomb per square meter | $\mathrm{C} / \mathrm{m}^{2}$ |
|  | Permittivity | farad per meter | F/m |
|  | Permeability | henry per meter | H/m |
|  | Exposure (X- and $\gamma$-rays) | coulomb per kilogram | $\mathrm{C} / \mathrm{kg}$ |


| TABLE 1.5 | Prefix | Symbol | Magnitude |
| :---: | :--- | :--- | :--- |
| Metric Prefixes. | yocto | y | $10^{-24}$ |
|  | zepto | z | $10^{-21}$ |
|  | atto | a | $10^{-18}$ |
|  | femto | f | $10^{-15}$ |
|  | pico | p | $10^{-12}$ |
|  | nano | n | $10^{-9}$ |
|  | micro | $\mu$ | $10^{-6}$ |
|  | milli | m | $10^{-3}$ |
|  | centi | c | $10^{-2}$ |
|  | deci | d | $10^{-1}$ |
|  | deka | da | $10^{1}$ |
|  | hecto | h | $10^{2}$ |
|  | kilo | k | $10^{3}$ |
|  | mega | M | $10^{6}$ |
|  | giga | G | $10^{9}$ |
|  | tera | T | $10^{12}$ |
|  | peta | P | $10^{15}$ |
|  | exa | E | $10^{18}$ |
|  | zetta | Z | $10^{21}$ |
|  | yotta | Y | $10^{24}$ |
|  |  |  |  |


| TABLE 1.6 | Name | Symbol | Value in SI Units |
| :---: | :---: | :---: | :---: |
| Units Outside the SI That Are Accepted for Use with the SI System. | Minute (time) | min | $1 \mathrm{~min}=60 \mathrm{~s}$ |
|  | Hour | h | $1 \mathrm{~h}=60 \mathrm{~min}=3600 \mathrm{~s}$ |
|  | Day | d | $1 \mathrm{~d}=24 \mathrm{~h}=86,400 \mathrm{~s}$ |
|  | Degree (angle) | - | $1^{\circ}=(\pi / 180) \mathrm{rad}$ |
|  | Minute (angle) | ' | $1^{\prime}=(1 / 60)^{\circ}=(\pi / 10,800) \mathrm{rad}$ |
|  | Second (angle) | " | $1^{\prime \prime}=(1 / 60)^{\prime}=(\pi / 648,000) \mathrm{rad}$ |
|  | Liter | L | $1 \mathrm{~L}=1 \mathrm{dm}^{3}=10^{-3} \mathrm{~m}^{3}$ |
|  | Metric ton | t | $1 \mathrm{t}=1000 \mathrm{~kg}$ |
|  | Neper | Np | $1 \mathrm{~Np}=20 \log _{10}(\mathrm{e}) \mathrm{dB}=20 / \ln (10) \mathrm{dB}$ |
|  | Bel | B | $1 \mathrm{~B}=(1 / 2) \ln (10) \mathrm{Np}, 1 \mathrm{~dB}=0.1 \mathrm{~B}$ |
|  | Electronvolt | eV | $1 \mathrm{eV}=1.60218 \times 10^{-19} \mathrm{~J}$ |
|  | Unified atomic mass unit | u | $1 \mathrm{u}=1.66054 \times 10^{-27} \mathrm{~kg}$ |
|  | Astronomical unit | ua | 1 ua $=1.49598 \times 10^{11} \mathrm{~m}$ |

[^0]A second is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

An ampere is the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newtons per meter of length.

A kelvin, is $1 / 273.16$ of the thermodynamic temperature of the triple point of water.
A mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12 ; its symbol is moll. When the mole is used, the elementary entities must be specified; they may be atoms, molecules, ion, electrons, other particles, or specified groups of such particles.

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz $(\mathrm{Hz})$ and that has the radiant intensity in that direction of $1 / 683$ watt per steradian.

### 1.3 Charge, Voltage, Current, and Power

### 1.3.1 ELECTRIC CHARGE

Atoms are the basic building blocks of matter. The nucleus of atoms consists of protons and neutrons. Electrons orbit around the nucleus. Protons are positively charged, and electrons are negatively charged, while neutrons are electrically neutral. The amount of charge on the proton is given by

$$
e=1.60217662 \times 10^{-19} C
$$

Here, the unit for charge is in coulombs (C).

$$
-e=-1.60217662 \times 10^{-19} C
$$

Notice that the charge is quantized as the integral multiple of $e$. Since there are equal numbers of protons and electrons in an atom, it is electrically neutral. When a plastic is rubbed by fur, some electrons from the fur are transferred to the plastic. Since the fur lost electrons and the plastic gained them, the former is positively charged and the latter negatively charged. When the fur and the plastic are placed close together, they attract each other. Opposite charges attract, and like charges repel. However, since the electrons and protons are not destroyed, the total amount of charge remains the same. This is called the conservation of charge.

### 1.3.2 ELECTRIC FIELD

According to Coulomb's law, the magnitude of force between two charged bodies is proportional to the charges $Q$ and $q$ and inversely proportional to the distance squared; that is,

$$
\begin{equation*}
F=\frac{1}{4 \pi \varepsilon} \frac{Q q}{r^{2}} \tag{1.1}
\end{equation*}
$$

Here, $\varepsilon$ is permittivity of the medium. The permittivity of free space, $\varepsilon_{0}$, is given by

$$
\begin{equation*}
\varepsilon_{0}=\frac{1}{4 \pi c^{2} 10^{-7}}(\mathrm{~F} / \mathrm{m})=8.8541878176 \times 10^{-12}(\mathrm{~F} / \mathrm{m}) \tag{1.2}
\end{equation*}
$$

Here, $c$ is the speed of light in the vacuum, given by $c=299,792,458 \mathrm{~m} / \mathrm{s} \approx 3 \times 10^{8} \mathrm{~m} / \mathrm{s}$. The unit for permittivity is farads per meter $(\mathrm{F} / \mathrm{m})$. The direction of the force coincides with the line connecting the two bodies. If the charges have the same polarity, the two bodies
repel each other. On the other hand, if the charges have the opposite polarity, they attract each other.

If a positive test charge with magnitude $q$ is brought close to a positive point charge with magnitude $Q$, the test charge will have a repulsive force. The magnitude of the force is inversely proportional to the distance squared between the point charge and the test charge. The presence of the point charge creates a field around it, where charged particles experience force. This is called an electric field, which is defined as the force on a test charge $q$ as the charge $q$ decreases to zero; that is,

$$
\begin{equation*}
\boldsymbol{E}=\lim _{q \rightarrow 0} \frac{\boldsymbol{F}}{q} \quad(\mathrm{~V} / \mathrm{m}) \tag{1.3}
\end{equation*}
$$

The electric field is a force per unit charge. The electric field $\boldsymbol{E}$ is a vector quantity whose direction is the same as that of the force. Figure 1.1 shows the electric field for a positive point charge and charged parallel plates.

## FIGURE 1.1

Electric field for
(a) a point charge and (b) parallel plates.

(a)

(b)

If an object with charge $q$ is placed in the presence of electric field $\boldsymbol{E}$, the object will experience a force as follows:

$$
\begin{equation*}
\boldsymbol{F}=q \boldsymbol{E} \tag{1.4}
\end{equation*}
$$

For a positive point charge $Q$, the electric field is given by

$$
\begin{equation*}
\boldsymbol{E}=\frac{1}{4 \pi \varepsilon} \frac{Q}{r^{2}} \boldsymbol{a}_{r} \tag{1.5}
\end{equation*}
$$

where $\boldsymbol{a}_{r}$ is a unit vector in the radial direction from the positive point charge $Q$. For parallel plates with area $S$ per plate, distance d between the plates, the electric field is constant within the plates and the magnitude of the electric field is given by

$$
\begin{equation*}
E=\frac{Q}{\varepsilon S} \tag{1.6}
\end{equation*}
$$

The direction of the field is from the plate with positive charges to the plate with negative charges, as shown in Figure 1.1(b).

### 1.3.3 VOLTAGE

If a positive test charge $d q$ is moved against the electric field created by a positive charge, an external agent must apply work to the test charge. Let $d w_{A B}$ be the amount of the work
needed to move the test charge from B (initial) to A (final). Here, $d w_{A B}$ is the potential energy in joules. Then, the potential difference between points $A$ and $B$ is defined as the work done per unit charge against the force; that is,

$$
\begin{equation*}
v_{A B}=v_{A}-v_{B}=\frac{d w_{A B}}{d q} \quad(\mathrm{~J} / \mathrm{C}) \tag{1.7}
\end{equation*}
$$

The unit for the potential difference is joules per coulomb, which is also called a volt (V):

$$
1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C}
$$

The potential difference between A and B is called voltage. The potential difference between points $A$ and $B$ is given by

$$
\begin{equation*}
v_{A B}=v_{A}-v_{B}=-\int_{B}^{A} \boldsymbol{E} \cdot d \ell \tag{1.8}
\end{equation*}
$$

The negative sign implies that moving against the electric field increases the potential. For a positive point charge $Q$ at origin with an electric field given by Equation (1.5), the potential difference between two points A and B with distances $r_{A}$ and $r_{B}$, respectively, from $Q$ is given by

$$
\begin{equation*}
v_{A B}=v_{A}-v_{B}=-\int_{r_{B}}^{r_{A}} \frac{1}{4 \pi \varepsilon} \frac{Q}{r^{2}} d r=-\left.\frac{Q}{4 \pi \varepsilon}\left(\frac{-1}{r}\right)\right|_{r_{B}} ^{r_{A}}=\frac{Q}{4 \pi \varepsilon}\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right) \mathrm{V} \tag{1.9}
\end{equation*}
$$

Notice that the integral of $1 / r^{2}$ is $-1 / r$. If $r_{B}$ is infinity, the potential difference is

$$
\begin{equation*}
v_{A B}=v_{A}-v_{B}=v_{A}=\frac{Q}{4 \pi \varepsilon r_{A}} \mathrm{~V} \tag{1.10}
\end{equation*}
$$

The potential is zero at infinity. This is a reference potential. For the parallel plates shown in Figure 1.1(b), the potential difference between A and B is

$$
\begin{equation*}
v=E d=\frac{Q}{\varepsilon S} d \tag{1.11}
\end{equation*}
$$

If the potential at B is set at zero $\left(v_{B}=0\right)$, the potential at point A is given by

$$
\begin{equation*}
v_{A}=\frac{d w_{A}}{d q} \quad(\mathrm{~J} / \mathrm{C}) \tag{1.12}
\end{equation*}
$$

or simply

$$
\begin{equation*}
v=\frac{d w}{d q} \quad(\mathrm{~J} / \mathrm{C}) \tag{1.13}
\end{equation*}
$$

The potential difference $v$ is called voltage. A battery is a device that converts chemical energy to electrical energy. When a positive charge is moved from the negative terminal to the positive terminal through the $12-\mathrm{V}$ battery, the battery does 12 joules of work on each unit charge. The potential energy of the charge increases by 12 joules. The battery provides energy to the rest of the circuit.

### 1.3.4 CURRENT

In the absence of an electric field, the free electrons in the conduction band of conductors such as copper wire make random movements. The number of electrons crossing a cross-sectional area of the copper wire from left to right will equal the number of electrons crossing the same cross-sectional area from right to left. The net number of electrons crossing this area will be zero. When an electric field is applied along the copper wire, the negatively charged electrons will move toward the direction of higher potential. The current is defined as the total amount of charge $q$ passing through a cross-sectional area in $t$ seconds; that is,

$$
\begin{equation*}
I=\frac{q}{t} \tag{1.14}
\end{equation*}
$$

The unit for the current is coulombs per second ( $\mathrm{C} / \mathrm{s}$ ) or amperes ( A ). If the amount of charge crossing the area changes with time, the current is defined as

$$
\begin{equation*}
i(t)=\frac{d q(t)}{d t} \tag{1.15}
\end{equation*}
$$

The direction of current is defined as the direction of positive charges. Since the charge carriers inside the conductors are electrons, the direction of electrons is opposite to the direction of the current. Figure 1.2 shows the directions of the electric field, current, and electron inside a conductor.

## FIGURE 1.2

The directions of $E, I$, and $e$.


The charge transferred between time $t_{1}$ and $t_{2}$ can be obtained by integrating the current from $t_{1}$ and $t_{2}$; that is,

$$
\begin{equation*}
q=\int_{t_{1}}^{t_{2}} i(\lambda) d \lambda \tag{1.16}
\end{equation*}
$$

## EXAMPLE 1.1

The charge flowing into a circuit element for $t \geq 0$ is given by

$$
q(t)=2 \times 10^{-3}\left(1-e^{-1000 t}\right) \text { coulomb }
$$

Find the current flowing into the element for $t \geq 0$.

$$
i(t)=\frac{d q(t)}{d t}=2 \times 10^{-3} \times 1000 e^{-1000 t} \quad A=2 e^{-1000 t} \quad A \text { for } t \geq 0
$$

## Exercise 1.1

The charge flowing into a circuit element for $t \geq 0$ is given by

$$
q(t)=4 \times 10^{-3} e^{-2000 t} \text { coulomb }
$$

Find the current flowing into the element for $t \geq 0$.
Answer:

$$
i(t)=\frac{d q(t)}{d t}=-8 e^{-2000 t} \quad A \text { for } t \geq 0
$$

## EXAMPLE 1.2

The current flowing into a circuit element is given by

$$
i(t)=5 \sin (2 \pi 10 t) \mathrm{mA}
$$

for $t \geq 0$. Find the charge flowing into the device for $t \geq 0$. Also, find the total charge entered into the device at $t=0.05 \mathrm{~s}$.

$$
\begin{aligned}
q(t) & =\int_{0}^{t} i(\lambda) d \lambda=\frac{5 \times 10^{-3}}{2 \pi 10}[1-\cos (2 \pi 10 t)] \\
& =7.9577 \times 10^{-5}[1-\cos (2 \pi 10 t)] \text { coulomb }
\end{aligned}
$$

At $t=0.05 \mathrm{~s}$, we have

$$
q(0.05)=1.5915 \times 10^{-4}[1-\cos (2 \pi 10 \times 0.05)]=1.5915 \times 10^{-4} \text { coulombs }
$$

## Exercise 1.2

The current flowing into a circuit element is given by

$$
i(t)=5 \cos (2 \pi 10 t) \mathrm{mA}
$$

for $t \geq 0$. Find the charge flowing into the device for $t \geq 0$. Also, find the total charge entered into the device at $t=0.0125 \mathrm{~s}$.

Answer:

$$
\begin{aligned}
& q(t)=\int_{0}^{t} i(\lambda) d \lambda=\frac{5 \times 10^{-3}}{2 \pi 10} \sin (2 \pi 10 t)=7.9577 \times 10^{-5} \sin (2 \pi 10 t) \text { coulombs } \\
& q(0.0125)=7.9577 \times 10^{-5} \sin (2 \pi 10 \times 0.0125)=5.6270 \times 10^{-5} \text { coulombs }
\end{aligned}
$$

## FICURE 1.3

(a) Power is positive.
(b) Power is negative.


### 1.3.5 POWER

The battery provides a constant potential difference (voltage) of $v$ volts from the negative terminal to the positive terminal. When a positive charge $d q$ is moved from the negative terminal to the positive terminal through the battery, the potential energy is increased by $d q v=d w$. When the positive charge $d q$ moves through the rest of the circuit from the positive terminal to the negative terminal, the potential energy is decreased by the same amount $(d q v)$. The rate of potential energy loss is given by

$$
\begin{equation*}
p=\frac{d w}{d t}=\frac{d q v}{d t}=i v \tag{1.17}
\end{equation*}
$$

The rate of energy loss is defined as power. Equation (1.17) can be rewritten as

$$
\begin{equation*}
d w=d q v=p d t \tag{1.18}
\end{equation*}
$$

The energy is the product of power and time. If Equation (1.18) is integrated as a function of time, we get

$$
\begin{equation*}
w(t)=\int_{-\infty}^{t} p(\lambda) d \lambda \tag{1.19}
\end{equation*}
$$

According to Equation (1.19), the energy is the integral of power. As shown in Equation (1.17), power is the derivative of energy. Taking the derivative of Equation (1.19), we obtain

$$
\begin{equation*}
p(t)=\frac{d w(t)}{d t} \tag{1.20}
\end{equation*}
$$

If the voltage and the current are time-varying, the power is also time-varying. If the voltage and current are expressed as a function of time, Equation (1.17) can be written as

$$
\begin{equation*}
p(t)=i(t) v(t) \tag{1.21}
\end{equation*}
$$

The power given by Equation (1.21) is called instantaneous power. According to Equation (1.21), instantaneous power is the product of current and voltage as a function of time. In the passive sign convention, if the direction of current is from the positive terminal of a device, through the device, and to the negative terminal of the device [as shown in Figure 1.3(a)], the power is positive. On the other hand, if the current leaves the positive terminal of a device, flows through the rest of the circuit, and enters the negative terminal of the device [as shown in Figure 1.3(b)], the power is negative.

If power is positive [i.e., $p(t)>0$ ], the element is absorbing power. On the other hand, if power is negative, the element is delivering (supplying) power. In a given circuit, the total absorbed power equals the total delivered or supplied power. This is called conservation of power.

EXAMPLE 1.3

Let the voltage across an element be $v(t)=100 \cos (2 \pi 60 t) \mathrm{V}$, and the current though the element from positive terminal to negative terminal be $i(t)=5 \cos (2 \pi 60 t)$ A for $t \geq 0$. Find the instantaneous power $p(t)$ and plot $p(t)$.

$$
\begin{aligned}
p(t) & =i(t) v(t)=5 \cos (2 \pi 60 t) \times 100 \cos (2 \pi 60 t)=500 \cos ^{2}(2 \pi 60 t) \\
& =250+250 \cos (2 \pi \times 120 t) \mathrm{W}
\end{aligned}
$$

The power $p(t)$ is shown in Figure 1.4. Since $p(t) \geq 0$ for all $t$, the element is not delivering power any time. On average, the element absorbs 250 W of power.


## Exercise 1.3

Let the voltage across an element be $v(t)=100 \cos (2 \pi 60 t) \mathrm{V}$ and the current though the element from positive terminal to negative terminal be $i(t)=6 \sin (2 \pi 60 t)$ A for $t \geq 0$. Find the instantaneous power $p(t)$ and plot $p(t)$.

$$
p(t)=i(t) v(t)=6 \sin (2 \pi 60 t) \times 100 \cos (2 \pi 60 t)=300 \sin (2 \pi 120 t) \mathrm{W} .
$$

The power $p(t)$ is shown in Figure 1.5. Since $p(t)>0$ half of the time and $p(t)<0$ the other half of the time, the element absorbs power for $1 / 240 \mathrm{~s}$, then delivers power for the next $1 / 240 \mathrm{~s}$, and then repeats the cycle. On average, the element does not absorb any power.

## FIGURE 1.5

Power $p(t)$.


### 1.4 Independent Sources

## FIGURE 1.6

Circuit symbols for voltage sources.

(a)
(b)

A voltage source with voltage $V_{s}$ provides a constant potential difference to the circuit connected between the positive terminal and the negative terminal. The circuit notations for the voltage source are shown in Figure 1.6.

If a positive charge $\Delta q$ is moved from the negative terminal to the positive terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_{s}$. If a negative charge with magnitude $\Delta q$ is moved from the positive terminal to the negative terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_{s}$. A battery is an example of a voltage source.

## FIGURE 1.7

A circuit symbol for the current source.


A current source with current $I_{s}$ provides a constant current of $I_{s}$ amperes to the circuit connected to the two terminals. The circuit notation for the current source is shown in Figure 1.7.

### 1.4.1 DIRECT CURRENT SOURCES AND ALTERNATING CURRENT SOURCES

If the voltage from the voltage source is constant with time, the voltage source is called the direct current (dc) source. Likewise, if the current from the current source is constant with time, the current source is called the direct current (dc) source.

If the voltage from the voltage source is a sinusoid, as shown in Figure 1.8, the voltage source is called alternating current (ac) voltage source. Likewise, if the current from the current source is a sinusoid, the current source is called alternating current (ac) current source. A detailed discussion of ac signals is given in Chapter 9. The circuit notation for an ac voltage source and ac current source are shown in Figure 1.9. The phase is given in degrees. The circuit notation for dc voltage shown in Figure 1.6(a) and the circuit notation for dc current shown in Figure 1.7 are also used for ac voltage and ac current, respectively.

## FIGURE 1.8

Plot of a cosine wave with period $T$, amplitude $V_{m}$, and phase zero.


## FIGURE 1.9

Circuit symbols for (a) ac voltage source; (b) ac current source.

## FIGURE 1.10



(a)

(b)

When dc voltage sources are connected in series, they can be combined into a single equivalent dc voltage source, as shown in Figure 1.10, where $V_{3}=V_{1}+V_{2}=4.5 \mathrm{~V}+7.5 \mathrm{~V}=12 \mathrm{~V}$. If there are other components, such as the resistors between $V_{1}$ and $V_{2}$ in the circuit shown in Figure 1.10, the voltage sources can be combined, so long as all the components are connected in series. Resistors are discussed further in Chapter 2.

When dc current sources are connected in parallel, they can be combined into a single equivalent dc current source, as shown in Figure 1.11 , where $I_{3}=I_{1}+I_{2}=3 \mathrm{~A}+5 \mathrm{~A}=8 \mathrm{~A}$. If other components such as resistors are connected in parallel to $I_{1}$ and $I_{2}$ in the circuit shown in Figure 1.11, the current sources can be combined, so long as all the components are connected in parallel between the same points.

## FIGURE 1.11

An equivalent current source.


## EXAMPLE 1.4

Redraw the circuit shown in Figure 1.12 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

## FIGURE 1.12

Circuit for
EXAMPLE 1.4.


Since $V_{1}$ and $V_{2}$ are part of a single wire, they can be combined into the single voltage source $V_{3}$. Since $V_{2}$ has the same polarity as $V_{1}$, the value of $V_{3}$ is given by

$$
V_{3}=V_{1}+V_{2}=5 \mathrm{~V}+3 \mathrm{~V}=8 \mathrm{~V}
$$

Since $I_{1}$ and $I_{2}$ are connected between the same points in the circuit, they can be combined into the single current source $I_{3}$. Since $I_{2}$ has the same polarity as $I_{1}$, the value of $I_{3}$ is given by

$$
I_{3}=I_{1}+I_{2}=3 \mathrm{~mA}+2 \mathrm{~mA}=5 \mathrm{~mA}
$$

The equivalent circuit, with one voltage source and one current source, is shown in Figure 1.13.

## Example 1.4 continued

FIGURE 1.13
A circuit with one current source and one voltage source.


## Exercise 1.4

Redraw the circuit shown in Figure 1.14 with one voltage source and one current source, without affecting the voltages across and currents through the resistors in the circuit.

## FIGURE 1.14

Circuit for EXERCISE 1.4.


## Answer:

The equivalent circuit with one voltage source and one current source is shown in Figure 1.15.

## FIGURE 1.15

A circuit with one current source and one voltage source.



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